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Pierre Kornprobst, Gilles Aubert. Explicit Reconstruction for Image Inpainting. [Research Report] RR-5905, INRIA. 2006. inria-00071360

**HAL Id: inria-00071360**

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

# *Explicit Reconstruction for Image Inpainting*

Pierre Kornprobst — Gilles Aubert

**N° 5905**

Avril 2006

Thème BIO

A large blue rectangle occupies the lower half of the page. Overlaid on the left side of this rectangle is a large, light grey, stylized letter 'R'. To the right of the 'R', the words 'rapport' and 'de recherche' are written in a white, italicized, serif font, stacked vertically. A horizontal grey brushstroke is positioned below the text.

*rapport  
de recherche*





## Explicit Reconstruction for Image Inpainting

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Thème BIO — Systèmes biologiques  
Projets Odyssée

Rapport de recherche n° 5905 — Avril 2006 — 18 pages

**Abstract:** Image inpainting refers to techniques which allow to fill in a gap  $\Omega$  given the intensities around it. In this paper, we focus on geometric image inpainting for which several PDE based models have been proposed. Most of them rely on a transport or/and a diffusion equation of the intensity inside  $\Omega$ . The direction of the reconstructed isophotes and the gray levels are generally computed in a coupled way. Instead, we propose in this paper a two step approach. First we estimate the tangents of the missing isophotes using a tensor field diffusion process. Then we simply recover the gray levels by integrating along integral curves of the tensors principal eigenvectors. Such an approach has very few parameters to tune and we illustrate its performance on several synthetic and real examples.

**Key-words:** image inpainting, variational approach, PDE, tensor diffusion

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## Une Approche Explicite pour l’Inpainting

**Résumé :** L’*inpainting* d’images correspond aux techniques de restauration d’images qui permettent de remplir un domaine  $\Omega$  dans une image, étant donné les intensités au bord du domaine. Dans ce rapport, nous considérons le cas des images géométriques, pour lesquelles plusieurs méthodes par équations aux dérivées partielles (EDP) ont été proposées. La plupart d’entre elles sont basées sur des équations de transport et/ou de diffusion des intensités à l’intérieur de  $\Omega$ . La direction des isophotes reconstruites et les niveaux de gris sont en général estimés de manière couplées. Dans ce papier, nous proposons une approche en deux étapes séparées. Tout d’abord nous estimons la direction des tangentes aux isophotes à reconstruire par une méthode de diffusion d’un champ de tenseurs. Ensuite, les niveaux de gris sont retrouvés en intégrant les trajectoires, qui correspondent aux courbes intégrales associées aux directions propres principales du champ de tenseur. Cette approche ne nécessite que très peu de paramètres à régler et nous illustrons ses performances sur plusieurs exemples synthétiques et réels.

**Mots-clés :** inpainting d’image, approche variationnelle, EDP, diffusion de tenseurs

## 1 Introduction

The goal of inpainting is to restore a damaged or corrupted image where part of the information has been lost. Such degradations of an image may have different origins like image transmission problems or degradations in real images due to storage conditions or manipulations. Inpainting may also be an interesting tool for graphics people who need to remove artificially some parts of an image such as some overlapping text or tricks used in special effects (see example Figure 1 (a)). In any case, the restoration of missing parts has to be done so that the final image looks unaltered for an observer who does not know the original image.

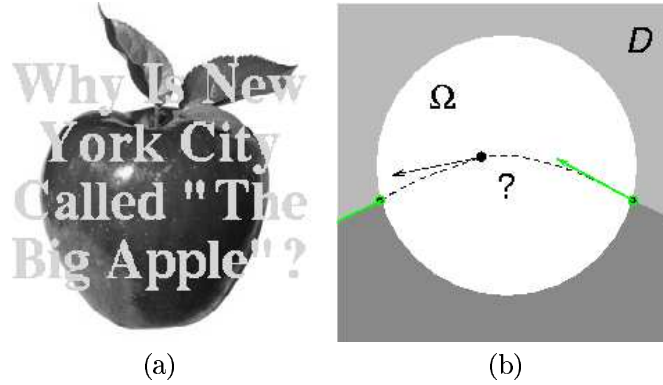


Figure 1: (a) Example of image with some text superimposed to be removed, (b) Synthetic image  $D$  with the part to be filled in  $\Omega$ . The dotted line represents the unknown level line.

Given a domain image  $D$ , a hole  $\Omega$  and an intensity  $u_0$  known over  $D - \Omega$ , we want to find an image  $u$  -an inpainting of  $u_0$ - that matches  $u_0$  outside the hole and which has meaningful content inside the hole  $\Omega$  (see Figure 1 (b)). Several kinds of approaches has been proposed in the literature to solve this problem depending on the application under consideration. In this paper we mainly focus on still geometric images, e.g. without fine texture contents and movement. We refer to [11, 6, 12] for inpainting algorithms dedicated to textured images and to [8, 17] (and references therein) for movies.

**Remark** Let us further comment on the dichotomy we mention between geometric and textured images. We claim that PDE-based approaches, which involve local diffusion operators, are suitable for geometric images and not for textured images. The main reason is that, by local diffusion, one may only prolongate the isophotes which are touching the hole: it is not possible to generate some closed isophotes inside the hole, i.e. textured patterns. This explains the recent success of non-local patch-based techniques which allow one to reconstruct also textures [11, 18]. Up to our knowledge, there is no pure PDE-based approach which

tackle any type of images. We refer the reader to the Appendix for more information and references.

For geometric images many PDE's or variational based approaches have been proposed in the last decade. Most inpainting PDE's methods make simultaneously the interpolation of the isophotes and the gray level intensities. In Section 2 we start by reviewing some of these related work. Section 3 presents our approach which is compounded of two steps. Several results on synthetic and real images are shown in Section 4. In Section 5 we conclude by suggesting future directions.

## 2 PDE-Based Inpainting Methods

The starting point of many PDE's contributions was the pioneering work by Nitzberg, Mumford and Shiota [16] who tried to identify occluding and occluded objects in an image (i.e. to identify T-junctions which are the points where the boundary  $\partial\Omega$  intersects the level lines of  $u_0$ ). Inspired from the work by Nitzberg et al. [16], Masnou and Morel proposed in [13] to join two compatible T-junctions  $T_1$  and  $T_2$  by a curve  $\Gamma$  (called a completion curve) lying in  $\Omega$  and minimizing the functional

$$\int_{\Gamma} (\alpha + \beta |\kappa|^p) dH^1 + (\theta_1, N_1) + (\theta_2, N_2),$$

where  $\alpha$  and  $\beta$  are positive constants,  $p \geq 1$ ,  $\kappa$  is the curvature of  $\Gamma$  and the last two terms denote the angles between the gradient of the image and the normal to  $\Gamma$  at the T-junctions  $T_1$  and  $T_2$ , respectively. Then by considering all the level sets and all admissible T-junctions, the global energy to minimize is of the form

$$\int_{-\infty}^{+\infty} \sum_{F_{\lambda}} \left( \int_{\Gamma} (\alpha + \beta |\kappa|^p) dH^1 + (\theta_1, N_1) + (\theta_2, N_2) \right) d\lambda,$$

where  $F_{\lambda}$  denotes the family of completion curves associated to the level set of  $u_0$ . Then they fill in the space between the completion curves with the appropriate gray level, the one at the boundary of the hole. Remark that this model is in fact a generalization of the Elastica model proposed in [16]. The main difference is that in [16] edges are considered rather than level lines, which yields the model dependent on the edge detector process.

In the same spirit, i.e., closely related to the work by Nitzberg et al. [16], Ballester et al. in a series of papers [1, 2, 3], proposed to solve the inpainting problem by considering a joint interpolation of the orthogonal direction  $\theta$  of level lines and of the gray levels. The vector field  $\theta$  is related to the image  $u$  according to the constraints  $\theta \cdot \nabla u = |\nabla u|$  and  $|\theta| \leq 1$ . Then they search for  $(u, \theta)$  as a minimizer of the variational problem

$$\min \int_{\Omega} |\operatorname{div}(\theta)|^p (a + b |\nabla k \star u|) dx,$$

$$\begin{aligned} |\theta| &\leq 1, \quad \theta \cdot \nabla u - |\nabla u| = 0 \text{ in } \Omega, \\ u &= u_0 \text{ in } B, \quad \theta \cdot N = \theta_0 \cdot N \text{ on } \partial\Omega, \end{aligned}$$

where  $p > 1$ ,  $a > 0$ ,  $b \geq 0$ ,  $k$  denotes a regularizing kernel of class  $C^1$  with  $k(x) > 0$  a.e. and  $N((x))$  denotes the outer unit normal at  $x \in \partial\Omega$ . The information is the value  $u_0$  on  $\partial\Omega$  and the vector field  $\theta_0$  of normals to the level lines of  $u_0$ .

There also exist many models which do not explicitly come from a variational principle. The first one by Bertalmio et al. [6], inspired from basic techniques used by art conservators, is based on the idea to propagate some information from the boundary of the hole inside the hole by using transport equations. In their model the authors want to propagate the value  $u_0$ , given on the boundary  $\partial\Omega$ , along the isophotes of the Laplacian (an edge detector) resulting the third order PDE:

$$\frac{\partial u}{\partial t}(x, y, t) = \nabla(\Delta u(x, y, t))^\perp \cdot \nabla u(x, y, t), \quad (1)$$

for every  $(x, y) \in \Omega$ , with the initial condition  $u(x, y, 0) = u_0^{ext}(x, y)$  where  $u_0^{ext}(x, y)$  is a continuous extension of the data  $u_0$  (which is only known in  $D - \Omega$ ) and  $u(x, y, t) = u_0(x, y)$  for all  $(x, y) \in \partial\Omega$  and for all  $t \geq 0$ . Notice that in their paper the authors did not use exactly the same equation as (1). On the right-hand side of (1) they wrote  $\nabla(\Delta u) \cdot \nabla u^\perp$  which has not the same meaning, but at convergence, i.e. when  $\frac{\partial u}{\partial t}(x, y, t) = 0$ , the result is the same since for all vectors  $V$  and  $W$  we have  $V^\perp \cdot W = -V \cdot W^\perp$ . To ensure a correct evolution of the direction field a diffusion process has to be interleaved with the inpainting process.

Many other approaches have been proposed (see also for example [19, 4, 10]), and we refer the reader to [5] for a general review on PDE-based inpainting methods.

From this review, we can observe the crucial role played by the isophotes. In the approaches [13, 16] related to the Elastica model, reconstructing them is the main focus. In other PDE-based approaches, in order to avoid this reconstruction, it has been proposed some diffusion transport processes on the gray-levels inside  $\Omega$ . In this paper we propose a method to estimate the tangent direction to the missing isophotes using a diffusion equation, and then reconstruct the gray levels explicitly. This is detailed in the next section.

## 3 Proposed Model

### 3.1 An Explicit Formulation

The contribution of this article is to give a direct formula for the inpainting problem. We suppose that a vector field  $V : \Omega \rightarrow \mathbb{R}^2$  corresponding to tangents to the isophotes is available. Then, for  $M = (x, y)$  in  $\Omega$ , we propose to define  $u(x, y)$  by

$$u(x, y) = u_0 \left( M + \int_0^{\bar{t}} V(X(t)) dt \right), \quad (2)$$



where  $u_0$  is the intensity which is known on  $D - \Omega$  (in particular on  $\partial\Omega$ ) and time  $\bar{t}$  is such that the point  $\left(M + \int_0^{\bar{t}} V(X(t))dt\right)$  belongs to  $\partial\Omega$  (see Section (3.3) for the precise definition of  $\bar{t}$ ).

This section presents the two steps of the model, namely the estimation of vector field compatible with tangent estimations on the boundaries (Section 3.2) and the filling in step done thanks to formula (6) (Section 3.3). The Figure 2 will help in illustrating the approach. Figure 2 (b) shows the original image where  $\Omega$  is compounded of three connected components.

### 3.2 Estimation of the Direction of the Isophotes

Knowing isophotes inside  $\Omega$  is equivalent with having their tangents at each point. In Figure 2 (c), tangent vectors to the isophotes at the boundary of  $\partial\Omega$  are represented. They are defined by  $n = \nabla u^\perp$ . For region 1, tangents are oriented in the same direction and propagating vectors inside  $\Omega$  will give a smooth vector field. For region 2, the situation is different and one may observe opposite directions at the boundaries, although directions (forgetting the orientation) are close. This discontinuity may appear to be artificial since it only comes from the intensity gray levels.

In order to avoid this phenomenon and only consider directions, the idea is to encode a vector by a symmetric definite positive matrix (also called tensor), which is used for instance in the tensor voting methodology [14]. For example, given a vector  $(n_1, n_2)$ , we define the following matrix

$$T = \mathcal{F}(n) = \begin{pmatrix} n_1^2 & n_1 n_2 \\ n_1 n_2 & n_2^2 \end{pmatrix}. \quad (3)$$

This matrix has only one non zero eigenvalue with eigenvector collinear to  $(n_1, n_2)$ , which suggests this correspondence between a vector and the associated tensor defined by (3). For a general matrix  $T$  in  $S^2$  (the set of symmetric non negative  $(2, 2)$  matrices), we can rewrite  $T$  using its eigenelements:

$$T = \lambda_{\max} v_{\max}^T v_{\max} + \lambda_{\min} v_{\min}^T v_{\min},$$

where  $\lambda_{\max} \geq \lambda_{\min} \geq 0$ , and  $v_{\max}$  orthogonal to  $v_{\min}$ . A standard way to represent such a matrix is by an ellipse, having its principal axes equal to the eigenvectors, scaled by the corresponding eigenvalues. The value  $\lambda_{\max} - \lambda_{\min}$  is often designated by the saliency, and it represents the "elongation" of the ellipse.

Given this representation, the problem is now to obtain a smooth tensor field in  $\Omega$  representing tangent directions to the missing isophotes. We assume that  $\Omega$  is compounded of a single connected component, otherwise the same approach will be applied for each connected component of  $\Omega$  independently. Let us introduce the sets

$$\begin{cases} \partial\Omega^{\text{Dir}} = \{(x, y) \in \partial\Omega / |\nabla u_0(x, y)| > 0\}, \\ \partial\Omega^{\text{Neu}} = \partial\Omega - \partial\Omega^{\text{Dir}}. \end{cases}$$

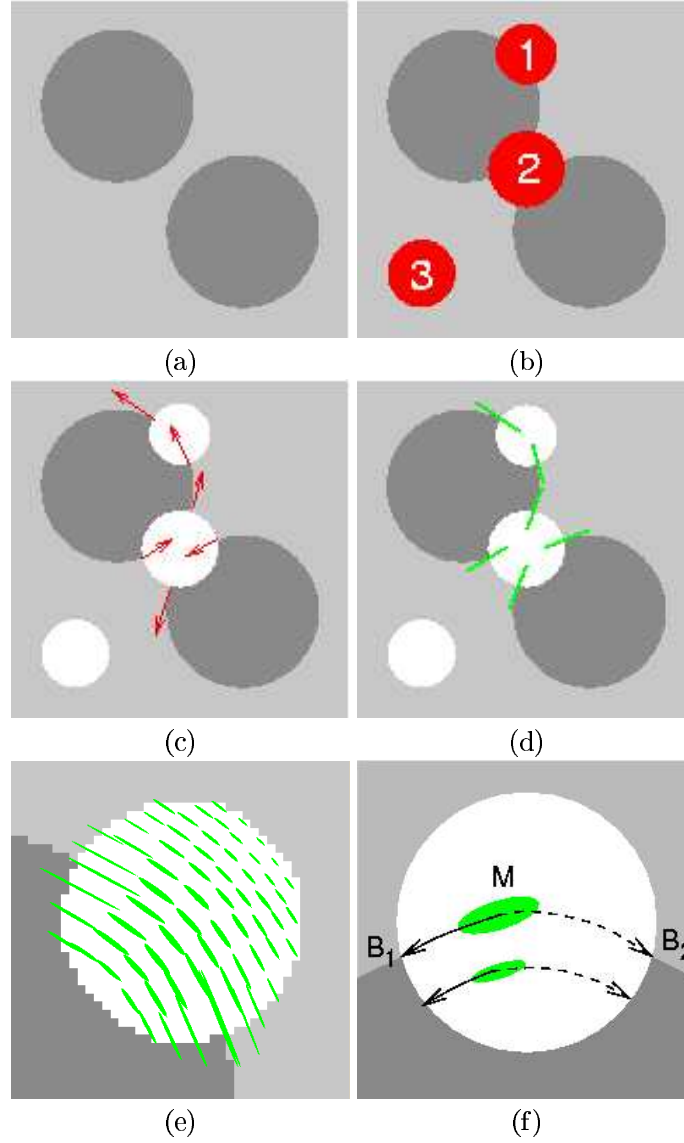


Figure 2: Some illustrations of the approach (a) Original image to be recovered, (b) Image with lost regions represented by red areas (i.e. the mask), (c) Image with tangents to the isophotes at the boundaries of the mask, (d) Same as (c) but directions are represented here by tensors (see equation (4)), (d) example on tensor field reconstructed with the equation (5), (e) Illustration for trajectory integration, see equation (6).

Case 1: If  $\partial\Omega^{\text{Dir}} = \emptyset$ , which corresponds to region 3 in Figure 2 (b), this means that  $|\nabla u_0(x, y)| = 0$  for all  $(x, y) \in \partial\Omega$ , or equivalently  $u_0 = \text{cte}$  on  $\partial\Omega$ . In such a situation, there are no need to estimate any isophotes and one may simply fill in the domain  $\Omega$  by this constant value.

Case 2: If  $\partial\Omega^{\text{Dir}} \neq \emptyset$ , we propose to obtain a smooth tensor field  $T$  using a diffusion process inside  $\Omega$ . One important issue is how to fix the boundary conditions (BC)? For  $(x, y)$  in  $\partial\Omega^{\text{Dir}}$  where some gradient information is present, Dirichlet BC are imposed, so that  $T$  will be defined by

$$T^{\text{Dir}}(x, y) = \begin{cases} \mathcal{F} \left( \frac{\nabla u_0^\perp}{|\nabla u_0^\perp|}(x, y) \right) & \text{if } |\nabla u_0|(x, y) > 0 \\ 0 & \text{(the null tensor), otherwise} \end{cases} \quad (4)$$

Remark that normalized gradients are used in the BC. By doing so, the reconstructed isophotes will not depend on the local gray level differences. This is illustrated in Figure 2 (d). Notice that matrix  $T$  defined by (4) has one null eigenvalue so that its representation is a "flat" ellipse. Homogeneous Neumann BC will be imposed over  $\partial\Omega^{\text{Neu}}$ .

In this paper, we propose to solve a simple heat equation applied on the matrices  $T$ , namely

$$\begin{cases} \frac{\partial T}{\partial t} = \Delta T & (x, y) \in \Omega, t > 0 \\ T(x, y, t) = T^{\text{Dir}}(x, y) & \forall t \geq 0, (x, y) \in \partial\Omega^{\text{Dir}}, \\ \frac{\partial T}{\partial n}(x, y, t) = 0 & \forall t, (x, y) \in \partial\Omega^{\text{Neu}}, \\ T(x, y, 0) = T^0(x, y), & (x, y) \in \Omega, \end{cases} \quad (5)$$

where  $T^0(x, y)$  is any matrix in  $S^2$  satisfying  $T^0(x, y) = T^{\text{Dir}}(x, y)$  for all  $(x, y) \in \partial\Omega^{\text{Dir}}$ , and  $\frac{\partial T}{\partial n}$  is the directional derivative along the normal to  $\partial\Omega$ . It can be proved that with initial and boundary conditions in  $S^2$ , the solution of (5) is in  $S^2$  for all  $t > 0$ . An example of result is shown in Figure 2 (e) where some reconstructed matrices are represented (it corresponds to region 1). It can be observed that matrices have a high saliency close to boundaries, and that their main direction is correctly smoothed.

**Remark** Contour completion is intuitively related to our approach but the major difference is that here we need to estimate directions everywhere, not only around contours. To illustrate this difference, we have done some experiments using the tensor voting framework developed by G. Médioni et al. [14]. The tensor voting is used to collect votes inside  $\Omega$ , where the tokens are the pixels with the tangent directions to the isophotes. Some qualitative results are shown in Figure 3. One advantage of tensor voting over a simple diffusion approach is that it takes into account the informations in a neighborhood of  $\Omega$  and also curvature informations (in the tensor fields). So for example, in the case of Figure 3 (a), we can expect that tensors direction will allow to reconstruct edges accurately, for the square and circle shapes. However, if we look more precisely to the directions of the tensors obtained in  $\Omega$ , then we observe that directions far from the salient edges are skewed in the

case of tensor voting (see Figure 3 (c)). This is not the case for a diffusion approach where directions are qualitatively more coherent with the directions of some isophotes (even if, in that case, we cannot define isophotes in the two homogenous regions).

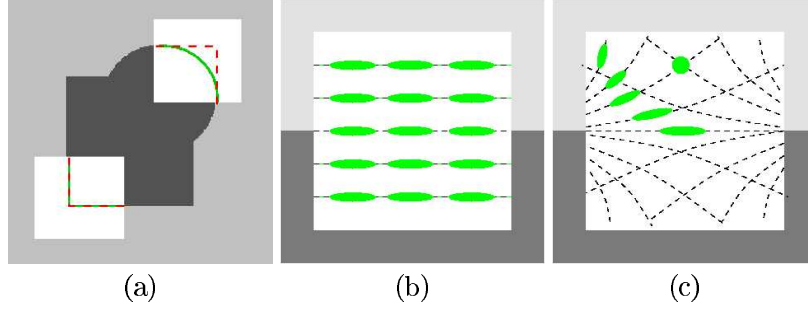


Figure 3: Pros and cons of a tensor voting approach to reconstruct the directions. (a) Illustration of the contour completion that could be obtained with tensor voting (green) and by diffusion (red dotted). (b) (resp., (c)) show some reconstructed tensors with the diffusion (resp., tensor voting) approach. In (c), dotted lines represent the directions of some tensor fields for a simple inpainting problem.

The next section shows how to simply exploit these directions to integrate intensities along isophotes, based on formula (2).

### 3.3 Integration of the Trajectories

In this section, we assume that there exists a dense tensor field  $T$  in  $\Omega$ , obtained after the diffusion process defined by (5). Since  $\partial\Omega^{\text{Dir}} \neq \emptyset$ , the diffusion process has then propagated the tangent directions from the boundaries. Given a point  $M$  in  $\Omega$ , we define the following trajectory  $X(t) = (x(t), y(t))$ :

$$\frac{\partial X}{\partial t}(t) = V(X(t)), X(0) = M, \quad (6)$$

where  $V(x, y)$  is collinear to the main eigenvector of  $T(x, y)$ . Let  $\bar{t}$  the shortest time such that the trajectory reaches the boundary. We suppose that such a time exists and in particular that (6) has no stationary point. Actually we suppose that the trajectory has a behavior as shown on Figure 2 (f) i.e. there exist two candidates, depending on the orientation chosen on the curve. Let us define the application  $\mathcal{G} : \Omega \rightarrow \partial\Omega$  which associates to a point  $M$  in  $\Omega$  the closest point in  $\partial\Omega$  obtained by integrating equation (6). Then we define, along the trajectory  $X(t)$ , the inpainting image inside  $\Omega$  by

$$u(X(t)) = \mathcal{G}(x, y) = u_0 \left( M + \int_0^{\bar{t}} V(X(t)) dt \right) \quad (7)$$

where  $u_0$  is the intensity which is known on  $\partial\Omega$ . Therefore  $u$  is constant along the trajectory. In particular  $u(M) = u(X(0))$  and

$$\frac{d}{dt}u(X(t)) = \nabla u(X(t)) \cdot V(X(t)) = 0 \text{ for all } t,$$

and for  $t = 0$  we get

$$\nabla u(M) \cdot V(M) = 0.$$

This equation means that the isophotes of the image reconstructed by (7) are tangent to the vector field  $V$ .

**Remark** We draw reader's attention that though we have obtained a transport equation our reconstruction method is not a characteristic method.

## 4 Experiments

### 4.1 Implementation Details

One interest of this approach is that it is simple to implement and does not require fine tuning of parameters. In fact, there are very few of them. One just need to fix the degree of smoothing of the tensor field, i.e. the number of iterations of the heat equation (5). The trajectory integration step is here implemented using a simple Euler method. Higher order method such as Runge-Kutta one's could also be considered to have more precise results.

### 4.2 Results on Synthetic and Real Images

The Figure 4 first shows the result obtained with the example already presented on Figure 2. Once the homogeneous region number 3 has been treated (see (b)), tensors can be estimated. They are represented in Figures (c) and (d). Orientation is described using a color code. The color at a pixel  $M$  with the tensor  $T(M)$  is obtained by starting from the middle of the image, following the main eigenvector of  $T(M)$  until reaching the boundaries of  $D$ , where color code is displayed. Another representation is given in (e) where some tensors inside region 2 are shown. Figure (f) is the final result after the integration step.

In order to test the model, we propose in Figure 5 to evaluate the approach on several situations, different angles, fine structures, triple junction. Good results are obtained, including for triple junctions.

Finally, Figures 6–8 show some results obtained on real images. They prove that very satisfying results can be achieved with this simple two-step approach.

## 5 Conclusion

The contribution of this paper to inpainting methods is to propose an alternative to gray level diffusion like approaches. Based on an explicit formula, we show that inpainting can

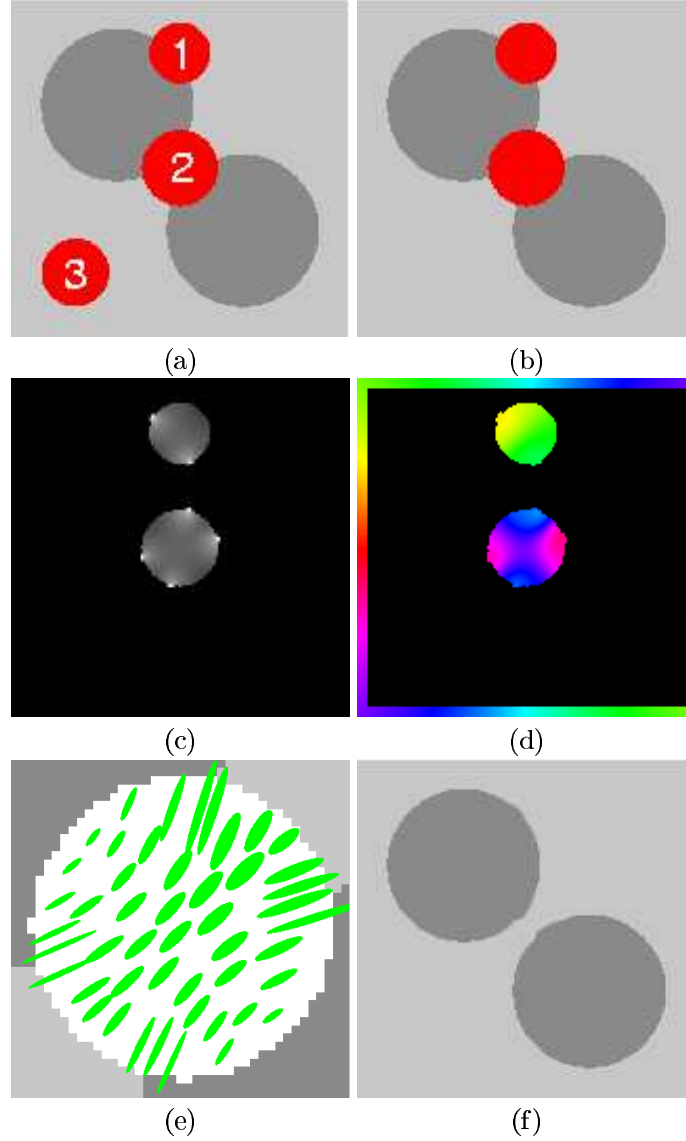


Figure 4: (a) Original image with  $\Omega$  consisting of three areas, (b) Result after preprocessing of homogeneous regions, i.e. connected components such that  $\partial\Omega^{\text{Dir}} = \emptyset$ , (c) and (d) represent matrices obtained after the heat equation, namely their saliency and orientation, (e) is a close-up on region 2 representing some of the matrices, (f) is the reconstructed image.

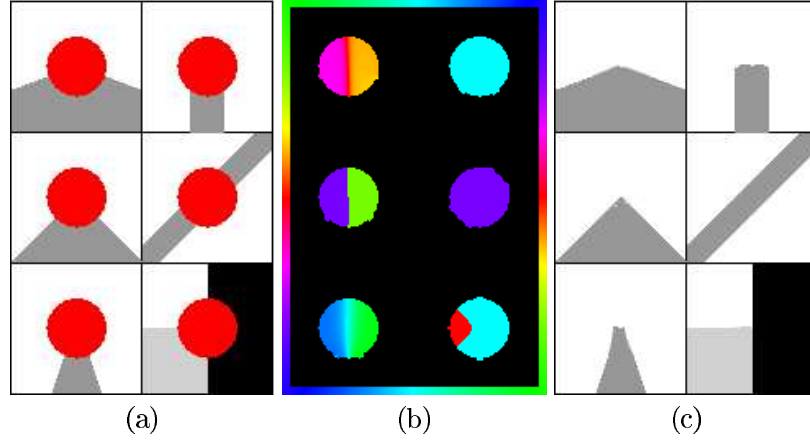


Figure 5: (a) Original test image with different typical situations, (b) matrices orientations obtained after the diffusion step, (c) is the reconstructed image.

be performed explicitly, once a direction of missing isophotes is available. Such an approach has very few parameters to be tuned and the two steps of the method do not involve any complex calculations.

In this work, the simplest PDE diffusion equation is used to estimate isophotes directions. It might be interesting to investigate if nonlinear operators could give better results. Another idea to evaluate the tensors inside the domain would be to use the tensor voting framework.

As far as theoretical matters are concerned, up to our knowledge, there are no justifications on existing models. We hope that such a simple formulation will allow us to prove existence and uniqueness of the solution. Some concerns such as the smoothness of the solution or the integrability of the trajectories (in particular the existence of time  $\bar{t}$ ) will need to be addressed.

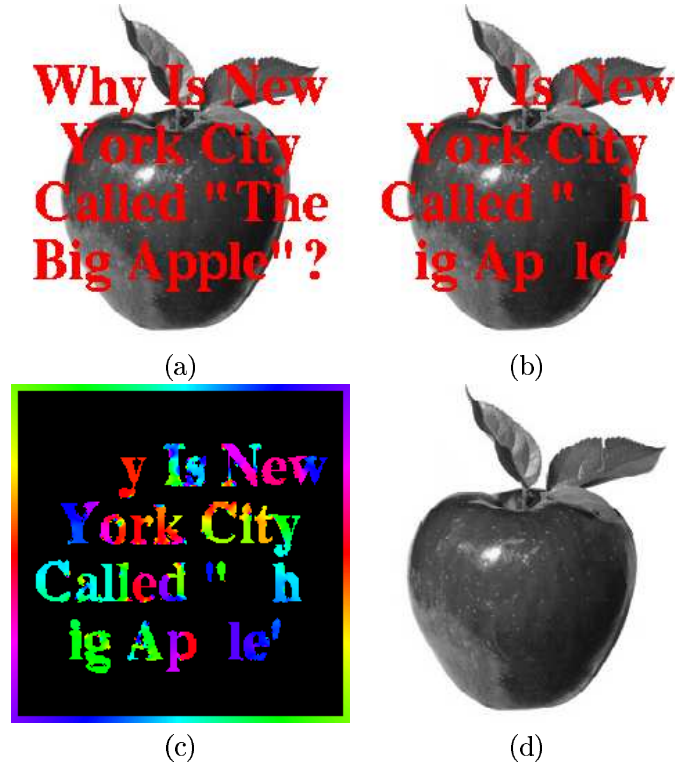


Figure 6: Two real examples. (a) Original images with mask superimposed in red, (b) result after preprocessing of homogeneous regions, i.e. connected components such that  $\partial\Omega^{\text{Dir}} = \emptyset$ , (c) orientation of the tensors obtained after the diffusion process, (d) inpainting result.



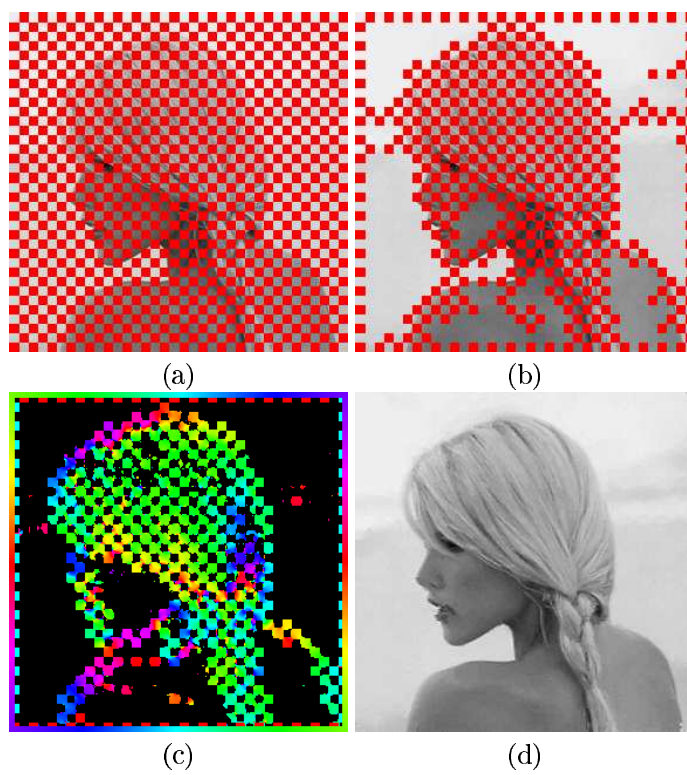


Figure 7: Same caption as in Figure 6

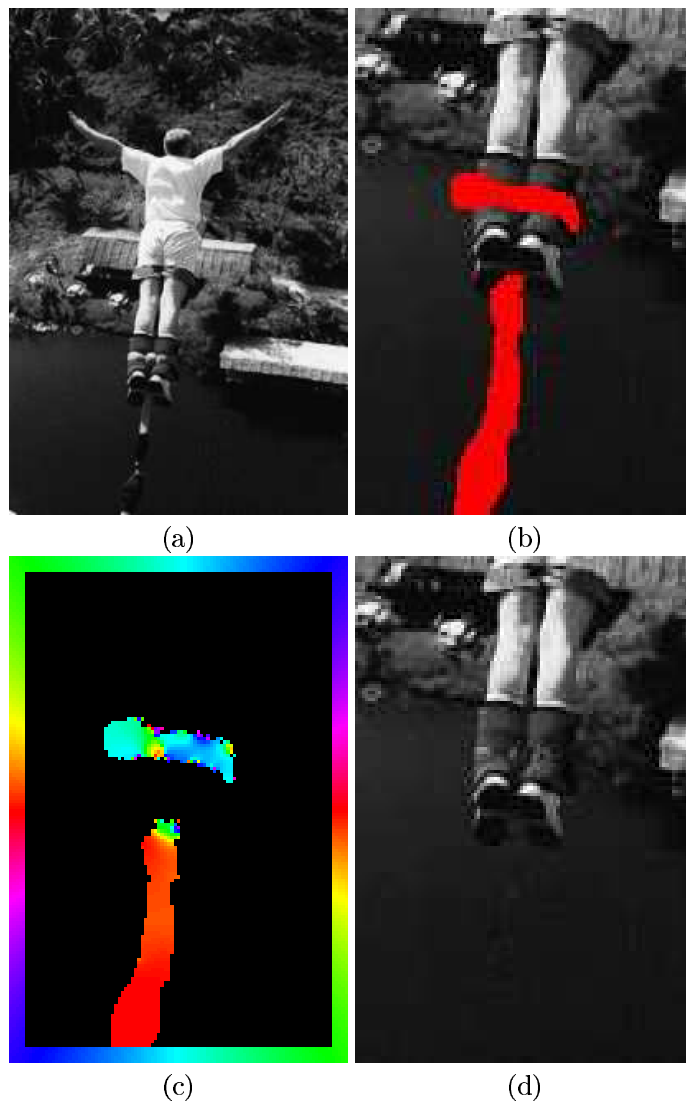


Figure 8: (a) Original image, (b) close-up on the lower part of the image with the mask shown in red color, (c) orientation of the matrices after diffusion, (d) final result.

## A Intrinsic Limitations of PDE-based Methods for Inpainting

In this report we have focused on geometrical images, as opposed to images with texture. We claimed that PDE-based approaches, which involve local diffusion operators, are suitable for geometric images and not for textured images. The main reason is that, by local diffusion, one may only prolongate the isophotes which are touching the hole: it is not possible to generate some closed isophotes inside the hole, i.e. textured patterns. This explains the recent success of non-local patch-based techniques which allow one to reconstruct also textures. In this Appendix we present some related work on textured image inpainting.

One kind of PDE-based method which tackles any image, uses clearly this dichotomy between geometric and textured images. For example, in the variational framework, Bertalmio et al. [7] propose to use some recent ideas by Meyer [15]. Given an image  $f$ , we can find  $u$  and  $v$  such that

$$f = u + v,$$

where  $u$  is well-structured and models homogeneous regions, and  $v$  contains oscillating patterns like texture and noise. Then the geometric part can be restored using some standard inpainting approaches (for instance [6]) and the texture part can be recovered using texture synthesis algorithms (for instance [12]). The reconstructed image is obtained by summing again both components.

Another difficulty is related to the size of the region to be filled in. Here again, it is unlikely that only diffusing some information in the hole will give satisfying results. Reinventing the content is also here very closely related to texture synthesis (see for example [12]). In recent work by Criminisi et al. [11], the authors proposed an algorithm that allows one to fill in an image progressively by looking for closest prototypes in a dictionary. More precisely, given a hole, a set of patterns around the hole is extracted to build a dictionary. Then, given a set of pixels, this set is filled in with the best-matching pattern. Note that the interest in considering patterns instead of working at the pixel level is that it speeds up the process and also adds some coherence in the reconstruction. Although the image filling in itself is not related to PDE-based inpainting, there is an underlying "priority" map, which drives the order of the filling process, that is diffused from the boundary inward and makes use of both geometric and photometric gradient-based information.

The approach [11] does not have any direct link with PDEs but this idea of looking for the best match between two patterns in order to select an intensity can be found in the nonlocal filter model by Buades et al. [9]. The nonlocal filter model uses the similarity between two pixels  $x$  and  $y$  defined by the similarity of the intensity gray level between their neighborhoods. The new intensity in  $x$  will be estimated thanks to values  $y$  such that their corresponding neighborhoods best match.

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INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)  
<http://www.inria.fr>  
ISSN 0249-6399